

**Directly Defining the inverse Mapping Method for Solving a Delay Differential Equation of Fractional Order**

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**Abstract**

In recent years, fractional differential equations and their applications have gotten extensive attention. The main reason is due to the rapid development of the theory of fractional calculus which is widely used in mathematics, physics, and other various disciplines. In this study, we applied a semi-analytical method called the Method of Directly Defining the inverse Mapping (MDDiM) to obtain a series solution for a delay differential equation of fractional order. A delay differential equation is a differential equation where the time derivatives at the current time depend on the solution and possibly its derivatives at previous times. Also, delay differential equations (DDEs) are used to introduce concepts arising in studies of infinite-dimensional dynamical systems. Liao introduced the MDDiM for nonlinear single ordinary differential equations. Recently, we extended the MDDiM to obtain approximate-analytical solutions of nonlinear single partial differential equations and systems of partial differential equations. In this work, we applied the MDDiM for acquiring an approximate-analytical solution of a fractional-order delay differential equation. Here, we obtained MDDiM solutions by considering the first three terms of the series solution. The best values of the convergence control parameter were determined by minimizing the square residual error of the approximate series solutions for different fractional orders. Further, it is investigated that the three term approximate series solution was accurate up to five decimal places, and agreed very well with the exact solutions and the solutions obtained by the Homotopy Analysis Method.

**Keywords:** *Delay differential equation of fractional order, fractional order differential equations, method of directly defining inverse mapping, series solution*

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